Analysis of Forward Error Correction and Achievable Rates for Optical Fiber Systems

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ABSTRACT

We study lower bounds on mutual information that are achievable rates for optimal and sub-optimal hard-decision (HD) and soft-decision (SD) decoding. These rates represent the maximum amount of information we can convey over a memoryless channel with a fixed input if ideal forward error correction (FEC) is employed. We find that the gain of complex SD decoding over HD decoding can be very small, depending on the modulation format and the system parameters. We further study the gap of a practical FEC scheme to its asymptotic limit.
Keywords: Achievable rates, mutual information, forward error correction, optical communication systems.

1. INTRODUCTION

The increasing demand for higher data rates in optical fiber networks drives the research into higher spectral efficiencies of such systems [1]. Besides using the available spectrum more effectively, the amount of information transmitted per channel use can be increased by high-order modulation formats, such as quadrature phase-shift keying (QPSK) or quadrature amplitude modulation (QAM). These advanced modulations are more susceptible to noise and nonlinearities, mainly because their minimum Euclidean distance is reduced. When transmission distances must be kept constant, the amount of distortions interfering with the signal during transmission is also constant, which requires stronger forward error correction (FEC) in order to keep the bit error rate (BER) after decoding at typically below 10^{-15}. The previously prevailing FEC with hard-decision (HD) decoding [2, 3], for simplicity referred to as HD FEC in this work, is replaced by a soft-decision (SD) FEC [4] that uses information about the reliability of a decision in its decoding process. For these SD FECs, mutual information (MI) is a relevant figure of merit as it gives the maximum amount of information that can be conveyed over a channel when an ideal, i.e., capacity-achieving, FEC is in place.

In this paper, we formally introduce lower bounds on MI of a channel with unknown statistics. Lower bounds for SD and HD decoding are compared for the additive white Gaussian noise (AWGN) channel. This analysis is extended to optical fiber simulations. Finally, the maximum transmission distance of a practical coding scheme is compared to its asymptotic limit.

2. MUTUAL INFORMATION AND ACHIEVABLE RATES

Although the fiber-optical channel has memory due to dispersion and nonlinear interference [5], correlations are often neglected in practical receivers that operate on a symbol-by-symbol basis. The MI $I(X;Y)$ is the largest achievable information rate for a memoryless channel with independent input X. It is defined as

$$I(X;Y) = \sum_{x\in\mathcal{X}} P_X(x) \int_{\mathbb{C}} p_{Y|X}(y|x) \log_2 \frac{p_{Y|X}(y|x)}{p_Y(y)} \mathrm{d}y, \tag{1}$$

where X has the distribution $P_X(x)$ on $\mathcal{X}$, $\mathbb{C}$ is the set of complex numbers, and $p_{Y|X}(y|x)$ denotes the channel transition probability. For an optical fiber channel, the channel $p_{Y|X}(y|x)$ is not known analytically. By assuming an auxiliary distribution $q_{Y|X}(y|x)$ instead of the true (yet memoryless) $p_{Y|X}(y|x)$, we obtain a lower bound on MI that is achievable for a decoder that takes the received symbols y to be distributed according to $q_{Y|X}(y|x)$ [6, Sec. VI].

2.1. Soft-Decision Decoding

A lower bound of $I(X;Y)$ is denoted by $\mathrm{R_{SD}}$,

$$I(X;Y) \geq \mathrm{R_{SD}} \triangleq \sum_{x\in\mathcal{X}} P_X(x) \int_{\mathbb{C}} p_{Y|X}(y|x) \log_2 \frac{q_{Y|X}(y|x)}{q_Y(y)} \mathrm{d}y, \tag{2}$$

which is an achievable rate for the communication system depicted in Fig. 1 (top). In fiber optics, the auxiliary channel is often assumed as circularly symmetric Gaussian with variance σ^2 that is estimated offline. Under this

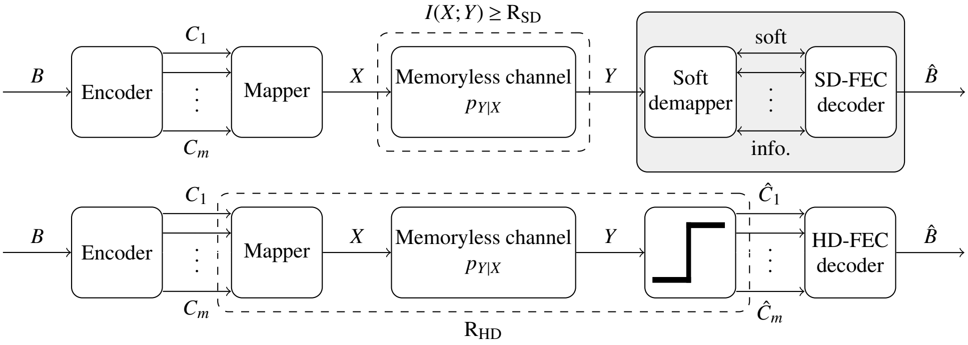

Figure 1: Block diagram of a communication system with SD decoding (top) and HD decoding (bottom).

assumption and for uniformly distributed input, i.e., $P_X(x) = \frac{1}{2^m}$ where m is the number of bits per input symbol, we compute R_{SD} in Monte Carlo simulations of N symbols as [7]

$$\mathrm{R}_{\mathrm{SD}} \approx m + \frac{1}{N}\sum_{n=1}^{N}\sum_{x\in\mathcal{X}} \frac{\exp\left(-\frac{(y_n-x)^2}{2\sigma^2}\right)}{\sum_{x'\in\mathcal{X}}\exp\left(-\frac{(y_n-x')^2}{2\sigma^2}\right)} \log_2 \frac{\exp\left(-\frac{(y_n-x)^2}{2\sigma^2}\right)}{\sum_{x'\in\mathcal{X}}\exp\left(-\frac{(y_n-x')^2}{2\sigma^2}\right)}. \tag{3}$$

The rate R_{SD} is achievable for a nonbinary FEC or for a receiver that allows iterations between demapper and decoder. If neither of these conditions is fulfilled, a commonly considered achievable rate is the sum of m bitwise MIs calculated from the soft input into the decoder. This quantity is sometimes referred to as the generalized mutual information (GMI) [8]. The GMI depends on the mapping between bits and symbols, and we consider Gray mapping in this work.

2.2. *Hard-Decision Decoding*

A drawback of an SD FEC is that processing soft information poses significant computational challenges on the receiver. A less complex alternative is to make hard decision on the received symbols as shown in Fig. 1 (bottom). Hard information, i.e., bits, are input into the HD decoder. We consider the case in which only one pair of encoder and decoder is used, yielding a practically relevant achievable rate for a coded modulation system with HD FEC and m parallel bit streams. We denote this rate by R_{HD} and define it as

$$\mathrm{R}_{\mathrm{HD}} \triangleq m\cdot(1-\mathrm{H}_{\mathrm{b}}(p)), \tag{4}$$

where $\mathrm{H}_{\mathrm{b}}(p) = -p\log_2 p - (1-p)\log_2(1-p)$ is the binary entropy function for the average BER p. Note that $\mathrm{R}_{\mathrm{SD}} \geq \mathrm{R}_{\mathrm{HD}}$ by the data processing inequality [9, Sec. 2.8]. In the following, we investigate the tightness of this bound for an AWGN channel.

2.3. *AWGN Analysis*

The achievable rates R_{SD} and R_{HD} for QPSK, 16-QAM and 64-QAM at different signal-to-noise ratios (SNRs) of an AWGN channel are shown in Fig. 2. We observe that at low SNR, the two rates are very close to each other. With increasing SNR, R_{SD} grows faster than R_{HD} as soft information is available at the receiver. For high SNR, both rates reach their modulation-dependent limit of m bits per symbol (bit/sym). We conclude that the difference between R_{SD} and R_{HD} strongly depends on the SNR, or equivalently, on the achievable information rate relative to m. We further observe that rate curves of R_{HD} intersect for the three modulation formats, while they do not cross if R_{SD} is considered.

Based on the analysis of Fig. 2, we conjecture a concave behavior of the rate difference $\mathrm{R}_{\mathrm{SD}} - \mathrm{R}_{\mathrm{HD}}$. To illustrate this, $\mathrm{R}_{\mathrm{SD}} - \mathrm{R}_{\mathrm{HD}}$ vs. SNR is shown in Fig. 3. Significant rate gains of 1 bit/sym for 64-QAM, more than 0.5 bit/sym for 16-QAM, and 0.25 bit/sym for QPSK are found. These rate gains can also be interpreted as rate losses due to employing less complex HD decoding. We extend this analysis to optical fiber systems in Section 4 and quantify the potential gain from using SD instead of HD decoding.

3. FORWARD ERROR CORRECTION

The analysis of Section 2 covers achievable rates, which means that ideal FEC with a block length going to infinity is assumed. To have a realistic comparison to these asymptotic limits, a practical concatenated FEC scheme that

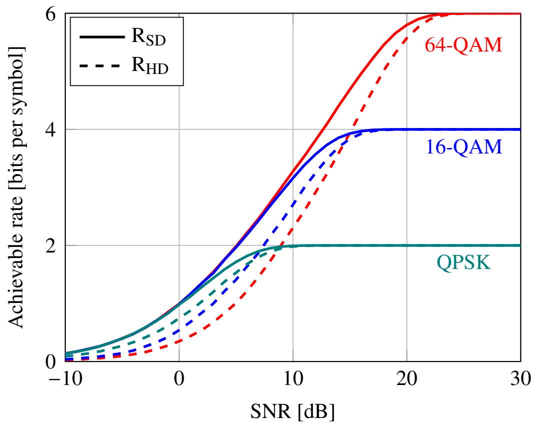

Figure 2: Achievable rates R_{SD} (solid) and R_{HD} (dashed) vs. SNR in dB for an AWGN channel.

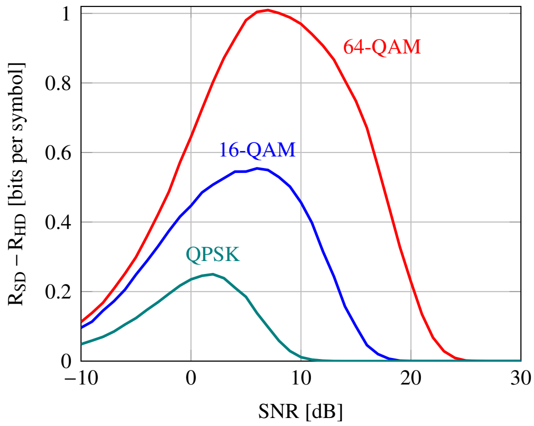

Figure 3: The rate difference $R_{SD} - R_{HD}$ vs. SNR in dB for QAM formats over an AWGN channel.

fulfills the BER requirement of 10^{-15} is considered. A low-density parity-check (LDPC) code from the DVB-S2 standard [10] of coding rate 3/4 (or equivalently a coding overhead (OH) of 1/3) is used as SD FEC. As this code exhibits an error floor before reaching a BER of 10^{-15}, a staircase code [11] is assumed as outer HD FEC to clean up the error floor at the expense of additional coding overhead.

4. NUMERICAL ANALYSIS OF OPTICAL FIBER SYSTEMS

4.1. Simulation Setup

A dual-polarization wavelength division multiplexing (WDM) system with 15 channels on a 30 GHz grid is considered. Modulation formats are QPSK, 16-QAM, and 64-QAM, each with a symbol rate of 28 GBaud. The pulse shape is root-raised cosine with 5% roll-off. The digital-to-analogue converter and the Mach-Zehnder modulator are assumed to be ideal. Fiber propagation is simulated with the split-step Fourier method, 32 samples per symbol and a constant step size of 100 m. The fiber link consists of spans of 100 km standard single-mode fiber, with an EDFA with 4 dB noise figure between spans. The fiber loss α is 0.2 dB/km, the nonlinear parameter γ=1.3 $(\mathrm{W{\cdot}km})^{-1}$, and the chromatic dispersion (CD) parameter D=17 ps/nm/km. At the receiver, the center channel is ideally band-pass filtered, down-sampled, and CD is compensated digitally. The achievable rates R_{SD} and R_{HD} are computed as shown in Eqs. (3) and (4), respectively. At most 50 iterations are used for LDPC decoding.

4.2. Simulation Results

For a fixed distance of 6000 km, achievable rates vs. launch powers per channel are depicted in Fig. 4 for three modulation formats. The difference between R_{SD} and R_{HD} is very small for QPSK at the optimum launch power. This indicates that using an SD FEC for QPSK gives only marginal gains over an HD FEC. Since both QPSK rates are close to their maximum of 2 bits per symbol, higher-order modulation should be used for larger spectral efficiencies. With 16-QAM, $R_{SD} \approx 2.9$ bit/sym and $R_{HD} \approx 2.4$ bit/sym, which is a rate increase over QPSK of 0.9 bit/sym and 0.5 bit/sym, respectively. Note that implementation penalties from using a high-order modulation are neglected in our simulations. Increasing the modulation format from 16-QAM to 64-QAM gives only a small gain in R_{SD} of 0.1 bit/sym. Since implementing 64-QAM in a practical transceiver is more challenging than 16-QAM, we conclude that 16-QAM is the best choice of modulation for SD decoding and the considered optical communication system with respect to a reasonable trade-off between spectral efficiency and complexity. We also note that R_{HD} for 16-QAM is 0.4 bit/sym larger than R_{HD} for 64-QAM, making 16-QAM the best among the examined formats for HD decoding.

In Fig. 5, the concatenated FEC scheme of Section 3 is compared to the asymptotic limit given by R_{SD}. We simulate 16-QAM over an increasing transmission distance and show the BER after LDPC decoding for every distance. By assuming a staircase code with 6.25% OH and perfect interleaving, the required BER of 10^{-15} is obtained as long as the post-LDPC BER is below $4.7{\cdot}10^{-3}$. For the considered setup, this is true for up to 51 spans. The information rate of 16-QAM with an LDPC code of rate 3/4 and an staircase FEC of rate $(1+0.0625)^{-1} \approx 0.94$ is 2.82 bit/sym. To find the maximum reach for this information rate, the transmission distance is increased in steps of 100 km and an achievable rate is computed for every distance until the rate drops below 2.82 bit/sym. A potential gain of 31%, from 51 to 67 spans, is found for R_{SD}. If the GMI is taken as figure of merit, the maximum distance

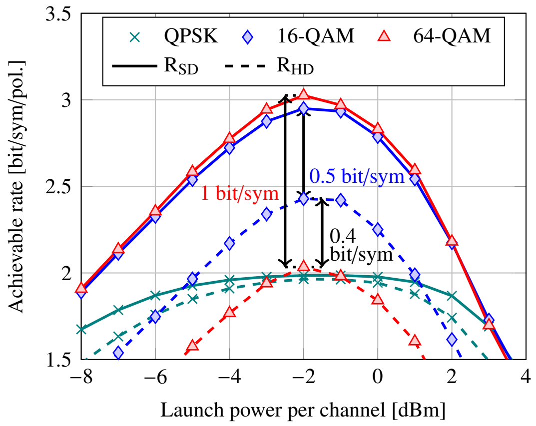

Figure 4: R_{SD} (solid) and R_{HD} (dashed) vs. launch power per channel of a fiber system with QPSK and QAM input (markers).

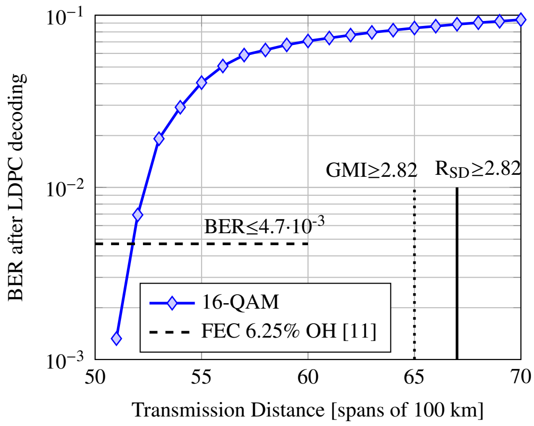

Figure 5: 16-QAM can be transmitted over 51 spans, while the asymptotic limit given by R_{SD} and GMI is 67 and 65 spans, respectively.

is 65 spans, which is 27% larger than the distance of the FEC scheme. The benefit of a capacity-achieving FEC can also be quantified when the distance is fixed. For the 51 spans of the LDPC and staircase scheme, R_{SD} is 3.12 bit/sym, which is a gain of 0.3 bit/sym. The GMI is 3.1 bit/sym at 51 spans. Besides the fact that the DVB-S2 LDPC code is not capacity-achieving, the reason for this gap is that a concatenated FEC, i.e. a combination of SD and HD decoding, is compared to R_{SD}, which assumes SD decoding only.

5. CONCLUSION

Achievable rates for hard and soft decoding are investigated. Using them as a figure of merit enables us to find an optimum modulation format for a specific optical fiber system. We further compare these achievable rates to a concatenated FEC scheme. For the considered LDPC and staircase FEC scheme, the maximum transmission distance can be increased by 31% by a capacity-achieving FEC. For a fixed distance, the gain in information rate is found to be 0.3 bit/sym. Quantifying the penalty from the LDPC error floor remains for future work. Also, LDPC codes that are highly relevant for optics, such as spatially coupled LDPC codes [12], could be investigated.

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