Information Quality (IQ) Factor as Soft-Decision Decoding Threshold for Optical Communications

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Abstract *A novel quantity called IQ-factor is proposed that facilitates the analysis of soft-decision decoding. The IQ-factor is based on mutual information and naturally relates to the Q-factor. We use IQ to define decoding thresholds. Simulations show that IQ is an accurate estimate of the system performance.*

Introduction

Optical long-haul networks continue to demand higher data rates that can be achieved by high-order modulation formats. Since optical channels are susceptible to distortions, strong forward error correction (FEC) is required to ensure that the bit error rate (BER) after decoding, denoted BER_{out} is typically in the range of 10^{-15}. Soft-decision (SD) decoding[1] gives the best performance and improves hard decisions (HD) that do not take into account the reliability of decisions. Hence, SD decoding is increasingly used in optical systems.

The Q-factor is a suitable quantity for HD decoding of algebraic codes in order to determine whether a certain BER_{out} can be reached. For iterative SD decoding, however, Q is not an adequate figure of merit as it does not take into account the additional information offered by soft decisions and thus does not reliably predict BER_{out}. In this paper, we introduce a new figure of merit for systems employing SD decoding and show that this figure of merit accurately predicts the performance of optical systems.

Mutual Information

Mutual information (MI) is a natural choice for a figure of merit in the context of SD decoding, especially for iterative decoding[2]. It expresses the statistical relations between the channel input and output in a single number that describes the rate of reliable communication for long codes such as those used for long-haul optical transmission. MI can be calculated from the likelihood ratios that are input to an SD decoder. For a formal definition, let X and Y be the input and output with respective realizations x and y of a discrete-input continuous-output memoryless channel. The bitwise MI $I^*(X;Y)$ of X and Y is defined as[3]

$$I^*(X;Y)=\frac{1}{M\log_2 M}\sum_{m=1}^{M}\int_{-\infty}^{\infty}p(y|x_m)\log_2\left[\frac{p(y|x_m)}{p(y)}\right]dy, \quad (1)$$

where M is the modulation order. Naturally, $I^*(X;Y)\in[0,1]$. Note that (1) is appropriate for memoryless channels. Although optical channels are not memoryless, neglecting memory and treating it as stochastic impairment gives a lower bound on the actual performance.

IQ-Factor

A new figure of merit for SD decoding should retain the useful properties of the Q-factor: the figure should be in dB, and it should be associable to the signal-to-noise ratio (SNR) of binary phase-shift keying (BPSK). Also, it should be easy to convert to and from Q. We propose to use the IQ-factor (IQ for information quality) defined as

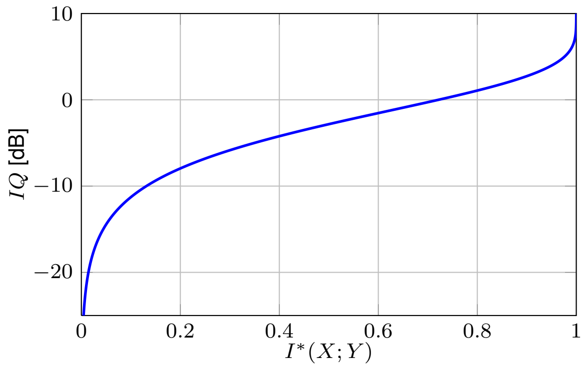

Fig. 1: IQ in dB vs. normalized MI $I^*(X;Y)$

$$IQ=20\log_{10}\left(\frac{J^{-1}\left(I^*\left(X;Y\right)\right)}{2\sqrt{2}}\right)\quad \text{in dB}, \quad (2)$$

where $J^{-1}(\cdot)$ denotes the inverse J-function[4]. $J^{-1}(I^*(X;Y))$ is defined as the variance of the L-values $L(Y)$ of Y distorted by circular symmetric (c.s.) additive white Gaussian noise (AWGN) with variance σ_n^2. For BPSK, this variance is $(2/\sigma_n)^2$. $(J^{-1}(I^*(X;Y))/2)^2$ is thus the ratio E_b/N_0 of energy per bit to noise power spectral density for BPSK and c.s. AWGN. The additional factor of $\sqrt{2}$ in the denominator of (2) is because E_b/N_0 is half the SNR for BPSK. As $I^*(X;Y)\leq 1$ for any modulation format, IQ is the SNR that would be required to achieve a particular $I^*(X;Y)$ for a c.s. AWGN channel if BPSK were used. Figure 1 depicts the relation between IQ and $I^*(X;Y)$. We see that $I^*(X;Y)\to 1$ already in the medium SNR regime and $1-I^*(X;Y)<10^{-5}$ for an SNR of 10dB. Hence for practical applications, IQ can be upper-bounded to 10dB.

Consider a system with equiprobable binary input symbols, hard decisions, and a symmetric channel with bit error probability p_b that corresponds to Q as per $Q=20\log_{10}\left(\sqrt{2}\cdot\text{erfc}^{-1}\left(2p_\text{b}\right)\right)$. IQ can be calculated from the Q-factor as

$$IQ=20\log_{10}\left(\frac{J^{-1}\left(1-H_\text{b}\left(\text{erfc}\left(\frac{10^{(Q/20)}}{\sqrt{2}}\right)/2\right)\right)}{2\sqrt{2}}\right) \quad (3)$$

with $\text{erfc}(\cdot)$ being the complementary error function, $\text{erfc}^{-1}(\cdot)$ its inverse, and $H_\text{b}(\cdot)$ the binary entropy function[3]. With both Q and IQ in dB, an approximation of (3) yields $IQ \simeq 1.07 \cdot Q - 4.66$. This linear fit matches (3) accurately for $-10\text{dB} \leq IQ \leq 10\text{dB}$.

Decoding Thresholds

A necessary and a sufficient condition for decoding is given by means of IQ. Let IQ denote the IQ-factor calculated at the decoder input according to (1) and (2). The underlying MI is calculated from the soft input to the decoder.

For a necessary decoding condition, information theory tells us that $I^*(X;Y)$ is the capacity C of a memoryless channel with discrete input that is distributed according to X, and that $C > R$ in order for any code of rate R to achieve an arbitrarily small BER[5]. Therefore, successful decoding is possible for any code of rate R if IQ is larger than IQ_{nec},

$$IQ > IQ_{\text{nec}} = 20 \log_{10} \left(\frac{J^{-1}(R)}{2\sqrt{2}} \right). \tag{4}$$

If $IQ \leq IQ_{\text{nec}}$, decoding is guaranteed to fail for any code. The condition of (4) is necessary for decoding but is also sufficient for ideal, i.e. capacity-achieving codes. Practical codes exhibit a gap from the asymptotic Shannon coding limit due to their non-random structure and finite block length. For them, decoding cannot be ensured by merely fulfilling (4).

For a sufficient condition, assume the performance of a certain code is known in terms of BER_{out} over IQ. To guarantee successful decoding for the considered code and channel model, IQ has to be at least IQ_{suff} required to achieve a desired BER_{out} denoted $\text{BER}_{\text{target}}$,

$$IQ \geq IQ_{\text{suff}}. \tag{5}$$

The BER_{out}-IQ-relation can be obtained by Monte-Carlo simulations. It is depicted in Fig. 2 for quadrature amplitude modulation with 16 points (16-QAM), c.s. AWGN and the DVB-S2 low-density parity-check (LDPC) code[6] with R=0.9 and at most 50 decoding iterations. IQ_{nec}=2.73dB for R=0.9 and IQ_{suff}=3.26dB corresponding to $\text{BER}_{\text{target}}$=$10^{-4}$ are shown as dashed green line and red circle, respectively.

The sufficient condition of (5) is valid only if the channel model used to evaluate BER_{out} over IQ is identical to the channel of the system under inspection. Channels other than the one evaluated possess a slightly different IQ_{suff}. However, one advantage of MI is that it is relatively robust to varying channel conditions when the soft input to the decoder is calculated correctly for the varying channel characteristics[2]. This beneficial property for optical systems whose channel cannot be described exactly by an analytical model is used in the following section to show that IQ is a reliable quantity to estimate the decoding limit. Another advantage of IQ, especially compared to Q is that IQ allows us to regard the physical communication system and the coding scheme as two distinct entities that can be optimized separately. If $IQ < IQ_{\text{nec}}$, optimizing the decoder will under no conditions give a small BER_{out}. As decreasing the code length and/or decoding complexity in general results in an increased IQ_{suff}, a weaker code may still produce the required BER_{out}, depending on how much larger IQ is than IQ_{suff} and by how much the decoder complexity or block length is reduced.

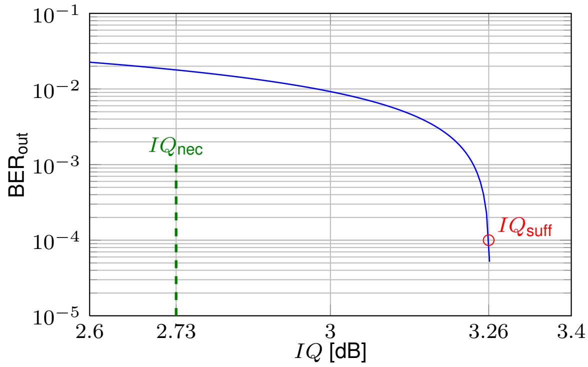

Fig. 2: IQ in dB required for moderate BER_{out} for c.s. AWGN, 16-QAM and the DVB-S2 LDPC code[6] with R=0.9.

Simulation Setup

A multi-span dual-polarization single-carrier wavelength-division multiplexing (WDM) system is simulated whose co-propagating WDM channels are spaced on a 50Ghz grid. The center channel is evaluated using a coherent receiver and digital signal processing (DSP) to examine how good the IQ estimate is. Parameters and subroutines that are not explicitly stated in this paper are identical to previous setups[7]. 2^{18} bits per polarization are created at the transmitter, encoded using the LDPC code of Fig. 2 and mapped onto 16-QAM symbols. After ideal digital-to-analog conversion, an IQ-Mach Zehnder modulator drives a laser at 1550nm with 100kHz linewidth to create the optical signal. The baud rate is 31.1Gbaud per polarization, pulses are shaped as non-return-to-zero.

Two different fiber setups are considered. The first one consists of spans of 80km standard single mode fiber (SMF) each followed by an erbium-doped fiber amplifier (EDFA). The second one additionally employs 16km of dispersion-compensating fiber (DCF) with attenuation 0.5$\frac{\text{dB}}{\text{km}}$, nonlinear coefficient 5.8$\frac{1}{\text{W}\cdot\text{km}}$ and dispersion -80$\frac{\text{ps}}{\text{nm}\cdot\text{km}}$ as well as another EDFA after each SMF-EDFA segment. Full dispersion compensation is achieved for both fiber setups, either electronically via DSP (no DCFs) or in-line (DCFs deployed).

The optical receiver front-end is a 90° optical hybrid with balanced photodiodes. After ideal analog-to-digital conversion, dispersion compensation (if not already performed optically), equalization and carrier phase recovery, the symbols are demapped, IQ is calculated from the soft input to the LDPC decoder and BER_{out} is determined. Perfect knowledge of the channel transition probabilities $p(y|x_m)$ of (1) is assumed.

Simulating a sufficiently high number of bit errors for $\text{BER}_{\text{target}}$=$10^{-15}$ would take several days even with real-time hardware. To present a proof of concept, the simulations are carried out at $\text{BER}_{\text{target}}$=$10^{-4}$. This approach is valid as long as the code under inspection

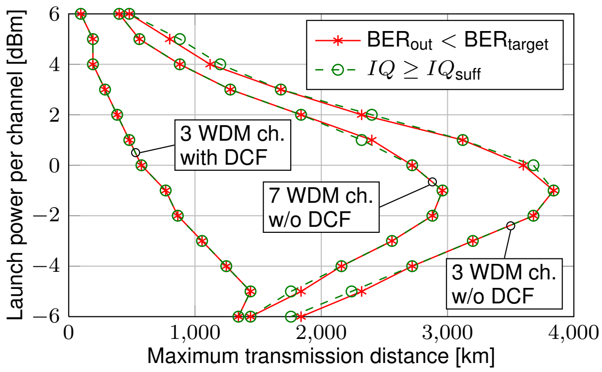

Fig. 3: Maximum reach for varying P_{tx}. Transmission is continued as long as BER$_{out}$ < BER$_{target}$ or $IQ \geq IQ_{suff}$.

does not have an error floor at the simulated BER. If an error floor exists, the LDPC code must be concatenated with a "clean-up" code that removes this error floor. The analysis must then be repeated for adapted IQ_{nec} and IQ_{suff}.

For R=0.9, IQ_{nec} equals 2.73dB according to (4). From Fig. 2 we see that IQ_{suff} is 3.26dB for BER$_{out}$=10^{-4} and c.s. AWGN. This channel model is used as a reference because the overall optical interference can be approximately modeled in many scenarios by a bivariate Gaussian distribution, either c.s. AWGN[8] or non-circular AWGN[9].

Simulation Results

Figure 3 depicts the maximum distance for a launch power per channel P_{tx} ranging from -6dBm to 6dBm in steps of 1dBm for 3 WDM channels with and without DCFs and for 7 channels without DCFs. The transmission distance is increased in increments of one span until a certain threshold is exceeded. For the solid red line with asterisks, the simulations are continued until BER$_{out}$ exceeds BER$_{target}$=10^{-4} for the first time. The maximum distance indicated by the dashed green line with circles is obtained when $IQ \geq IQ_{suff}$=3.26dB and transmission over an additional span would result in $IQ < IQ_{suff}$. This corresponds to the sufficient decoding threshold for c.s. AWGN. For the systems without DCFs, the general shape of the reach curves as well as the optimum launch power are in good agreement with previous results of long-haul 16-QAM simulations[7]. In comparison, the DCF system achieves a smaller reach and peaks at a smaller launch power because of the nonlinear DCFs, additional EDFAs, and the nonlinearities adding up coherently during propagation for full in-line compensation. For each of the three pairs of reach curves, IQ is an accurate estimate of the decoding performance determined by BER$_{out}$. The maximum estimation error is 1 span at distances of up to several thousand kilometers for all simulated launch powers from -6dBm to 6dBm and for all three systems.

Figure 4 depicts IQ (blue crosses) in dB for P_{tx} from -6dBm to 3dBm in steps of 1dBm after propagation of 3 WDM channels over 3200km (no DCF). The solid red and dashed green line show IQ_{suff}=3.26dB for BER$_{out}$=10^{-4} and IQ_{nec}=2.73dB for R=0.9, respectively.

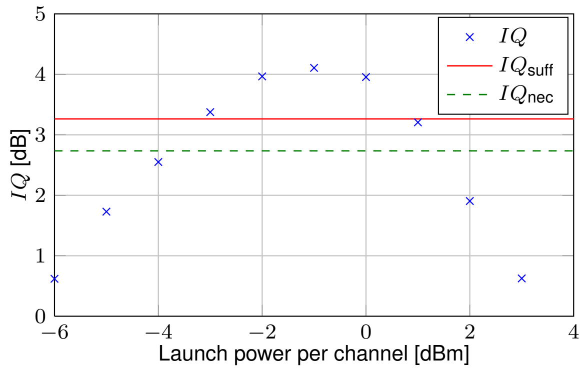

Fig. 4: IQ in dB after 40 spans (3 WDM channels, no DCF) vs. P_{tx}. Solid and dashed lines represent IQ_{suff} and IQ_{nec}.

For P_{tx} from -3dBm to 0dBm, $IQ \geq IQ_{suff}$ and BER$_{target}$ will most likely be achieved (we see from Fig. 3 that BER$_{target}$ is in fact achieved for these powers). For P_{tx}=1dBm, $IQ_{nec} < IQ < IQ_{suff}$. This means that decoding to BER$_{target}$ is in theory possible with a stronger coding scheme, but not with the used LDPC code. Decoding fails for P_{tx}≥2dBm or P_{tx}≤-4dBm because $IQ \leq IQ_{nec}$.

IQ for P_{tx} larger than 3dBm is omitted in Fig. 4 because the received signal is distorted so much that no accurate analysis is possible. The general distribution of IQ is as expected and in analogy to Fig. 3.

Conclusion

We have introduced the IQ-factor, a figure of merit based on MI and thus suitable for communication systems with SD decoding. A necessary decoding condition for any code and a sufficient decoding condition for a specific code and channel have been stated via IQ. We have also outlined that IQ can be used to facilitate the system design and optimization by separating the physical channel properties and the FEC scheme. Numerical simulations show that IQ corresponding to the decoding threshold obtained for a c.s. AWGN channel predicts the system performance very well for 16-QAM WDM systems with and without in-line dispersion compensation.

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